Kinematic subtleties in Einstein's first derivation of the Lorentz transformations

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We analyze Albert Einstein's derivation of the Lorentz transformations in his paper, ''Zur Elektrodynamik bewegter Körper,'' originally published in 1905. The analysis clarifies various misunderstandings in the secondary literature and reveals reasons why Einstein's work entailed interpretive difficulties.
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I. INTRODUCTION

As the centenary of the special theory of relativity approaches, readers may want to understand how Albert Einstein originally derived the Lorentz transformations.[1] On the one hand, we learn these equations easily as the simple algebraic core of Einstein's novel kinematics. On the other hand, Einstein's kinematics early on became known as notoriously difficult to understand, both to his peers and to nonspecialists. To help students and general readers today, the present paper clarifies kinematic subtleties that have deterred even specialists from understanding Einstein's derivation.

Einstein's first work on the relativity principle was difficult to understand for many readers. Consider the following examples.[2] In the spring of 1905, the physicist Josef Sauter was one of the first persons to hear about Einstein's work in depth. Sauter was one of Einstein's co-workers in the Swiss patent office, and because he had studied and published on Maxwell's theory of electromagnetism, Einstein ''gave him his notes, which Sauter criticized severely: 'I pestered him for a whole month with every possible objection'.'' Despite his criticisms, Sauter facilitated a meeting between Einstein and Paul Gruner, professor of theoretical physics at the University of Bern. Consequently, in 1907 Einstein gave his paper ''Zur Elektrodynamik bewegter Körper'' to Gruner, in support of his candidacy for a faculty appointment at Bern. In the words of Gruner, ''I received his essay although the whole theory at the time seemed to me to be highly problematical.'' The reaction of the faculty was even less favorable, as they ''declared the work as inadequate: it was more or less clearly rejected by most of the contemporary physicists.'' The professor of experimental physics, Aimé Forster, returned Einstein's paper with the remark, ''I can't understand a word of what you've written here.'' To be sure, a few physicists seem to have understood the gist of Einstein's arguments promptly, including Max Planck. But even Planck first wrote to Einstein asking that he clarify certain points in his paper.

What aspects of Einstein's work were problematic to early readers? Some difficulties were conceptual, because Einstein's ideas diverged radically from ordinary notions. Doubtless, the major difficulty was Einstein's novel concept of the relativity of time. But there also were mathematical subtleties. Difficulties in understanding Einstein's concepts have been discussed by many writers; in contrast, we analyze here his use of algebra.

Historians have remarked that the mathematics employed by Einstein was rather simple compared to the analytical methods then employed by leading theorists in electrodynamics. In the words of Russell McCormmach, Einstein ''was able to carry through a profound critique of the foundations of physics using elementary algebra, differential equations ... and with that light mathematical equipment he was able to formulate the kinematics of special relativity.''[3] But even though Einstein used only algebra and calculus in formulating his theory, his arguments were complex. The mathematical subtleties are best brought to light by reconstructing his derivations in detail. He summarized in only a few equations the results of hundreds of operations. His omission of intermediate steps may have obscured the intelligibility of his kinematics, as has happened with other scientific and mathematical texts throughout history.[4]

Because of its complexity, Einstein's first derivation has not been used in physics textbooks. In the *Annalen der Physik*, it occupies five pages, but once unraveled, it occupies approximately thirty pages of text. Einstein presented his derivation in about fifty-five equations, but worked out explicitly the derivation involves more than three hundred equations, consisting of roughly five hundred algebraic and differential operations. Einstein's succinct presentation thus allowed his readers to skip many details and to skim the substance of his argument, but at the expense of understanding his derivation clearly. For brevity, we will not give every step of Einstein's derivation. The present paper selects key points in need of clarification, and is thus meant to accompany a careful reading of Einstein's paper. Once the kinematic meaning of each term is understood, readers can then carry out the mathematical steps without doing so blindly.

What follows is an analysis of Section 3, ''Theory of Transformation of Coordinates and Times from a Resting System to a System in Uniform Translational Motion Relative to It,'' of Einstein's 1905 paper. The virtues of Einstein's first derivation have already been highlighted.[5] For example, Einstein derived the transformation equations without making any hypotheses about the constitution of matter, nor of intermolecular forces. The transformations were best suited for the solution of problems in electrodynamics, yet Einstein did not base them on the presumed validity of a contemporary electromagnetic theory, nor on Maxwell's equations. His derivation also was independent of whether light is presumed to consist of waves or particles, as Einstein used mainly the concept of light-ray, appropriate to both conceptions. Moreover, Einstein's derivation involved aspects that show both the logical economy of his thought as well as its conceptual roots. In particular, rather than postulating the invariance of the speed of light in nonaccelerated frames of reference, Einstein postulated the constancy of the speed of light in a single frame and then derived its invariance. Such expository devices paved the transition from a physics based on the privileged reference frame of the ether to one based on observational and formal symmetries. Because such key virtues of Einstein's paper are well known, our goal is only to identify and clarify formal ambiguities.

The best known analysis of Einstein's paper is Arthur I. Miller's 1981 study,[6] but it suffers, like others, from some errors that need correction. The present elucidation also will expose difficulties that Einstein's contemporaries may have experienced in attempting to understand his work.

II. KEY ASPECTS OF EINSTEIN'S DERIVATION

Compared to the conceptual analysis of measurement procedures in the first two sections of Einstein's paper, his formal derivation of the transformation equations was far more abstract. It followed the mathematical tradition of J.-L. Lagrange, S.-F. Lacroix, and others who dispensed with geometrical diagrams. It followed the descriptive approach espoused by Gustav Kirchhoff rather than explanatory approaches involving models of causes or mechanisms. It involved a profound reliance on formal requirements such as linearity, symmetry, and the theory of functions. It did not involve the methods or concepts of vector theory, although they had been advocated by influential physicists, such as Peter Guthrie Tait, Oliver Heaviside, and August Föppl, to replace the cumbersome methods of ''Cartesian'' coordinates, especially in electromagnetic theory.

The end result of Section 3 of Einstein's paper was a group of four equations. To obtain them, Einstein began by positing two Cartesian coordinate systems, K and k, with rectangular axes X, Y, Z, and Ξ, H, Z, respectively. He identified these systems with rigid bodies, each consisting of three mutually perpendicular rods, as he argued that the meaning of coordinates, lengths, and times should be given by specifications pertaining to rigid bodies and clocks. To distinguish the systems, he identified K as ''resting'' and k as ''moving'' (in quotation marks). He then derived the four transformations relating the position and time coordinates, x, y, z, t, of any physical event in K to the coordinates ξ, η, ζ, τ, of the same event in k. The transformations expressed the simplest relation between systems in relative motion: the case in which the axes of K are parallel to the axes of k, and the two systems move relative to one another in a straight line with a uniform speed v. By letting K and k be displaced only along the X and Ξ axes, the equations he found were

$$\tau = \beta(t - vx/V^2), \tag{1a}$$

$$\xi = \beta(x - vt), \tag{1b}$$

$$\eta = y, \tag{1c}$$

$$\zeta = z, \tag{1d}$$

where $\beta = 1/\sqrt{1 - v^2/V^2}$, and V is the speed of light in empty space, which we will henceforth designate by c, as already done in 1905 by some physicists. (For ease of reference to the 1905 paper, all other symbols are in Einstein's original notation.)

These four equations replaced the equations previously used by physicists to transform coordinates between systems in uniform rectilinear relative motion:

$$\tau = t, \tag{2a}$$

$$\xi = x - vt, \tag{2b}$$

$$\eta = y, \tag{2c}$$

$$\zeta = z. \tag{2d}$$

These equations were seldom stated explicitly; in particular, there was no need to express the equation relating the time coordinates, because time was assumed to be the same in all reference systems regardless of relative motions. Before the 1880s, physicists used a single variable t for all such systems. The other three equations, by contrast, were stated explicitly at least occasionally, as done by Lorentz, for example, in his 1886 paper ''De l'influence du mouvement de la terre sur les phénomènes lumineux.''[7] Due to his research on relative motion in optics and electromagnetics, he advanced a series of modifications to the traditional transformations that eventually led to the equations advocated by Larmor, Poincaré, Einstein, and others.[8] Hence, Poincaré gave the name ''Lorentz transformations'' to these new equations, although Woldemar Voigt had published equivalent equations in 1887.[9] In 1909 the simpler and older transformation equations were named the ''Galilean transformations'' by Philipp Frank.[10] What distinguished the new transformations in Einstein's work in comparison to the equivalent equations in the earlier work of other physicists was that Einstein introduced such transformations by means of general kinematic arguments, rather than introducing them exclusively for the solution of problems in optics and electrodynamics.

For simplicity, Einstein derived the four transformation equations given only the relative motion of the two systems along the X axis and Ξ axis. Thus, only the relation between the coordinates x and ξ, and between t and τ, would be expected to vary. To visualize the systems in relative motion, we may suppose that at the initial time t their coordinate axes coincide. Einstein began: ''First of all it is clear, that the equations must be *linear* on account of the properties of homogeneity that we attribute to space and time.''[11] This requirement can be expressed by the following:

$$\tau = a_{11}x + a_{12}y + a_{13}z + a_{14}t, \tag{3a}$$

$$\xi = a_{21}x + a_{22}y + a_{23}z + a_{24}t, \tag{3b}$$

$$\eta = a_{31}x + a_{32}y + a_{33}z + a_{34}t, \tag{3c}$$

$$\zeta = a_{41}x + a_{42}y + a_{43}z + a_{44}t. \tag{3d}$$

That is, each coordinate of k is a function of the coordinates of K and four undetermined constants. At any one time any particular value of a coordinate in K corresponds to only one value of a coordinate in k (otherwise, had Einstein allowed the equations to be quadratic, then to each coordinate value of K there could correspond two coordinate values of k, as, for example, $\eta = y^2$ yields two possible values for η).

By the homogeneity of space and time, Einstein presumably meant that no locations or directions in physical space are distinct or privileged, and that time likewise has a certain uniformity. For instance, if one were to place two identical measuring rods end to end, anywhere in space and at any time, all observers would agree that the result would be twice the length of one such rod. All points fixed in one nonaccelerated system move with the same velocity relative to another nonaccelerated system, and the transformations between systems must be indifferent to the choice of origin for either system.[12] In any case, Einstein did not offer such specific arguments to justify the linearity of the transformation equations.

Einstein proceeded: "We set

$$x' = x - vt, \tag{4}$$

so it is clear that to a point resting in the system k belongs a definite system of values x', y, z, independent of time."[13] Equation (4) is identical to the Galilean transformation $\xi = x - vt$.[14] The primed notation, as in x', was used occasionally by some writers to designate coordinates in an additional coordinate system.[15] So it might seem that Einstein had introduced a third coordinate system, because he used x' instead of ξ for the coordinate corresponding to x. Accordingly, in his analysis of Einstein's paper, Miller argued that Einstein had introduced a third set of coordinates, an "intermediate Galilean system," in addition to $K(x,y,z,t)$ and $k(\xi,\eta,\zeta,\tau)$. Miller claimed that Einstein related the coordinates in K and k "through an auxiliary set of space and time coordinates in $k(x',y'(=y),z'(=z),t'(=t))$ whose spatial portion x', y, z transforms according to the Galilean transformations; but every time coordinate is relativistic."[16] Likewise, Roberto Torretti claimed that Einstein introduced "an auxiliary coordinate system," although Torretti argued that it would be wrong to describe it "as a Galilei coordinate system."[17] Despite such claims, there is hardly anything in Einstein's paper to indicate that he introduced such an auxiliary system. Einstein did not refer to such a system, nor did he introduce the terms y', z', t'. Moreover, he used the term x' alongside the terms x, y, c, v, and t, which he repeatedly identified as values of system K, as he emphasized, for example, that t "always denotes a time of the resting system." Einstein did introduce, explicitly, a third coordinate system later in the paper. But only Eq. (4) can be interpreted as suggesting that a third system was involved from the outset.

The interpretation of x' as indicative of a third system creates difficulties for the validity of Einstein's arguments. Above all, how could Einstein expect to *derive* new transformation equations on the basis of a traditional transformation? This problematic question is of basic importance for understanding the transition from classical kinematics to Einstein's kinematics. Also, why would Einstein assume the validity of his principle of the constancy of the speed of light only *midway* through his derivation? Moreover, why would he tacitly assume outright that $y' = y$ and $z' = z$, without explanation, whereas he then spent pages demonstrating that $\eta = y$ and $\zeta = z$, rather than likewise assuming these relationships? Finally, toward the end of his derivation, when Einstein explicitly did introduce the terms x', y', z', t' as coordinates of a system, he used them as the coordinates of a system *identical* with K. So why would he use this very notation if he had first used it to designate coordinates of system k?

Such problems disappear once we realize that Eq. (4) need not be interpreted as a transformation equation, and that it may have an entirely different meaning. Specifically, it can describe the uniform motion of an object departing from a coordinate x', and moving along the X axis of K with a velocity v, such that it is located at a coordinate x at time t. This relation is a basic equation in kinematics: $x = x' + vt$, sometimes stated as $x = x_0 + ut$, as for example in Kirchhoff's *Mechanik*[18] (a work studied by Einstein), where it designated the rectilinear motion of a point just before Kirchhoff discussed the subject of transformation equations.[18] In this context, Einstein's use of Eq. (4) may be reinterpreted. He wrote: "We set $x' = x - vt$, so it is clear that to a point resting in the system k belongs a definite system of values x', y, z, independent of time." This "point resting in the system k" moves at a velocity v in K; it would depart from the position x', y, z, and reach the position x, y, z, after a time t, thus the value x would vary with time, whereas x' would not. Thus Eq. (4) can be understood as describing the uniform rectilinear motion of a point along the X axis of K. In this sense it does not imply a transformation between reference frames. Yet this interpretation, too, is defective, as we will see.

In any case, the point "resting" in k is described in terms of the values x', y, z of the system K. But why did Einstein not describe the position of the moving point with the values x, y, z instead? By employing the value x' instead of x, he could hope to simplify his derivation of the function $\tau(x',y,z,t)$, because then all terms, x', y, z, are constant, that is, "independent of time."[13] Could Einstein have obtained the same transformation equations had he used x instead of x' throughout? This question will be answered shortly.

To ascertain the form of the transformation equations, Einstein began by seeking the relation between t and τ. He wrote: "We first determine τ as a function of x', y, z, and t. To this end, we have to express in equations, that τ is nothing other than the embodiment of the data of clocks resting in system k, which have been synchronized according to the rule given."[19] Thus he based his derivation of the transformation equations on his procedure for synchronizing clocks: "From the origin of the system k a light ray was sent out at the time τ_0 along the X axis toward x', and from there at the time τ_1 is reflected back to the coordinates-origin, where it arrives at the time τ_2; thus it must then be: $\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$." Note that because Miller interpreted x' as a coordinate of k, he stated that Einstein's X in the passage just quoted "was an oversight," and that it should have been Ξ instead, "for consistency."[20] But if we interpret x' otherwise, it is not necessary to change Einstein's X; the light ray emitted from the origin of k is described with respect to K.

To relate the values of the function τ to the coordinate values of system K, Einstein expressed τ_0, τ_1, and τ_2 in terms of the corresponding values of x, y, z, and t of K:

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,t + \frac{x'}{c-v} + \frac{x'}{c+v}\right)\right]$$
$$= \tau\left(x',0,0,t + \frac{x'}{c-v}\right). \tag{5}$$

To explain Eq. (5) we must clarify an ambiguity: the expressions $c - v$ and $c + v$ seem to indicate variations in the speed of light. Miller, for example, interpreted them in this way.[21] Yet Einstein's analysis led to the conclusion that the speed of light relative to k should be c. Thus we should rightly doubt that he could derive this conclusion by letting light in k propagate at speeds of $c - v$ and $c + v$. Likewise, these expressions are not speeds of light relative to the "resting" system, because Einstein specified from the beginning that the speed of light in K was c. So what do these expressions mean?

The question is answered by clarifying the algebraic analysis Einstein presented in Section 2 of his paper where he used

$$t_B - t_A = \frac{rAB}{c-v}, \tag{6a}$$

$$t'_A - t_B = \frac{rAB}{c+v}. \tag{6b}$$

Here are the apparent variations in the speed of light. Einstein obtained Eq. (6) by the following procedure. Consider a rod of length rAB moving relative to K. One clock is attached to end A of the rod and another to B. A light ray departs from clock A at its clock reading of t_A. It reaches clock B at its reading of t_B, and then it is reflected back and arrives at the first clock at its time t'_A. Because the rod is moving, there is a difference between the time light takes to travel from one end of the rod to the other and the return trip. According to an observer in the stationary system, the ray of light traveling toward B moves an extra distance $v(t_B - t_A)$ in addition to rAB, because the end-point B of the rod moves away as the light ray approaches. Upon reflection at B the ray travels a distance less than rAB because A approaches it. Hence the travel times of light in each direction are

$$t_B - t_A = \frac{rAB + v(t_B - t_A)}{c}, \tag{7a}$$

$$t'_A - t_B = \frac{-rAB + v(t'_A - t_B)}{-c}. \tag{7b}$$

Einstein didn't give these equations, but notice that the magnitude of the velocities of light in opposite directions is the same: c. By solving for each time interval, we obtain Eq. (6). Thus we see that the expressions $c-v$ and $c+v$ did not enter Einstein's argument as speeds of light relative to K or k, but as *the rates with which light approaches moving points relative to* K. That is, $c-v$ is the rate of light approaching clock B, and $c+v$ is its rate of approach toward clock A. Such expressions don't imply any transformation, but instead, the usual vectorial addition of velocities within a single frame, which is valid exactly in both traditional kinematics as well as in Einstein's. This important distinction between the addition of velocities in one system and the transformation of velocities between two systems is neglected often.[22]

Einstein used Eqs. (6) and (4) to establish the value of each time coordinate in Eq. (5). As stated, he needed to ascribe definite values to the coordinates in K of the three events: the incidence of a light ray on clock A, then on clock B, and then back to A, given that the two clocks are moving along the X axis. So we have:

$$\tfrac{1}{2}[\tau(x_0,y_0,z_0,t_0) + \tau(x_2,y_2,z_2,t_2)] = \tau(x_1,y_1,z_1,t_1). \tag{8}$$

Because Eq. (4) is valid for any time t, Einstein used the same symbol t to designate the arbitrary time of departure of the light ray from A, that is, he set

$$t_0 = t. \tag{9}$$

By letting $x' = rAB$, Eqs. (6a) and (6b) serve to establish the values of t_1 and t_2. The time t_0 corresponds to t_A, and likewise, $t_1 = t_B$. Thus Einstein found that the time of arrival of the light ray at B is

$$t_1 = t + \frac{x'}{c-v}, \tag{10}$$

and hence the total time taken by the light to return to clock A is

$$t_2 = t + \frac{x'}{c-v} + \frac{x'}{c+v}. \tag{11}$$

Miller's procedure[23] to obtain these results for t_0, t_1, and t_2 is somewhat different from the one here, because instead of using Einstein's equation (4), he used: $x = x' + vt_1$. Anyhow, given these results for t_0, t_1, and t_2, Einstein wrote Eq. (5). If we expect Eq. (5) to agree with Eq. (8), then it should give the values of the function τ in terms of the values of the coordinates in K for the three events in question. However, Eq. (5) does not agree with Eq. (8).

Consider again the process of synchronization of moving clocks as observed from the "resting" system K. One clock is attached to the origin of system k, and the other clock is a distance away on the Ξ axis, such that both clocks move uniformly with k, along the X axis of K. If the coordinate systems coincide at the instant when the light ray is emitted, the ray departs from the origin of both K and k. It then travels to the distant clock and is reflected back to the first. It thus returns to the origin of k, because this process aims to synchronize the moving clocks. Because the origin of k has moved away from the origin of K, then the x coordinate of the returning light ray cannot be 0 in K. Thus it is problematic to expect that $x_2 = 0$ is the coordinate value in K for the location of the ray on its return. Likewise, if we interpret Einstein's x' as the location of the ray on its arrival at clock B, we also run into difficulties, because the light ray did not travel only a distance x' to B, but an additional distance because the clock moved away.

In short, the problem is that whereas the values given for the time coordinates t_0, t_1, t_2 are values referred to K, the values for the coordinates x_0, x_1, x_2 seem to refer to k. Normally, a statement of coordinates in parentheses, such as (x_1,y_1,z_1,t_1), was understood to designate values of the coordinates in a *single* system. In contrast, Einstein seems to have mixed values from two different systems. This ambiguity poses interpretive difficulties. We can eliminate them by disregarding x' and determining $\tau(x,y,z,t)$ strictly in terms of x, as well will demonstrate in the Appendix. At this point, however, there is still one way out that makes sense of Eq. (4), namely to give x' another interpretation, making it stand neither for a coordinate of the moving system nor for the fixed initial coordinate of a moving point.

Suppose that at the initial time t_0, the origins of K and k do not coincide, but instead are separated by a distance vt_0. In this case, if we let x_A represent the varying position of clock A, we have:

$$x_A - vt = x'_A = 0. \tag{12}$$

That is, for any time t we define a constant, $x'_A = 0$, that describes the separation of clock A from the moving origin of k along the X axis of K. (Because clock A is attached to the origin of k, that constant is 0.) Likewise, if x_B represents the varying positions of clock B:

$$x_B - vt = x'_B = x'. \tag{13}$$

That is, for any time t we define a constant $x'_B = x'$ that describes the constant separation of clock B from the moving origin of k. Accordingly, for the times of emission of the light ray from A, its arrival at B, and its return back to A, we have:

$$x_A - vt_0 = x'_A = 0, \tag{14a}$$

$$x_B - vt_1 = x'_B = x', \tag{14b}$$

$$x_A - vt_2 = x'_A = 0. \tag{14c}$$

The values 0, x', 0, serve to explain the first terms in each parentheses in Eq. (5). This explanation implies that we may understand Eq. (4) as meaning that $x'_s = x_{\text{clock}} - vt$, where x'_s is any constant expressing the separation (in K) between any fixed clock on k and the origin of k at any given time. Thus, Einstein's definition of x' as describing a point at rest in k with a value independent of time, should refer not just to one point but to any point fixed on k, in particular, to the two points where the clocks are attached.[24] Hence, to be more precise about his notation, Einstein could have stated that he sought a function $\tau(x'_s, y, z, t)$, rather than just $\tau(x', y, z, t)$ because the latter corresponds only to the events at clock B. Likewise, his statement that the light ray travels "along the X axis toward x', and from there" is reflected back, should not be interpreted literally as meaning that x' is the coordinate of the light ray when it arrives at clock B, because that coordinate is actually $x_B = x' + vt_1$. Rather, to make sense of his expressions, we have to understand him as meaning to say that the light ray is reflected from the constant point of k where clock B is fixed.

Note that to interpret the text consistently, we have gone a slight distance away from giving it a completely literal reading. Only by doing so can we arrive at a consistent explanation of Einstein's analysis. Hence we may surmise that the subtle ambiguities involved may have caused some difficulties in the understanding of his 1905 argument, especially for readers not sufficiently careful in kinematics, and perhaps for anyone who didn't have a good sense beforehand of the end result of the derivation.

By using differential analysis, Einstein next ascertained the dependence of τ on x', y, z, and t. We will not review his rather long procedure here, because the reader may consult Miller's study or Einstein's own paper. It is sufficient to say that his use of differential analysis was an unnecessary detour because he could have ascertained the τ function by means of linear algebra alone. In any case, he arrived at:

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right), \tag{15}$$

explaining that "a is a function $\varphi(v)$ presently unknown, and where for brevity it is assumed that at the origin of k, for $\tau=0$ is $t=0$."[25] Although his words seem to suggest that $a = \varphi(v)$, he did not quite use this relation, but instead, $a = \varphi(v)b$, as we will see (he should have just stated that "a is a function of v"), but this was just a minor oversight. Einstein's analysis also leads to:

$$\xi = a\frac{x'}{1 - v^2/c^2}, \tag{16a}$$

$$\eta = a\frac{y}{\sqrt{1 - v^2/c^2}}, \tag{16b}$$

$$\zeta = a\frac{z}{\sqrt{1 - v^2/c^2}}. \tag{16c}$$

He simplified Eq. (16) by finally "Substituting for x' its value," namely, $x' = x - vt$, as stated at the outset, so that

$$\xi = a\frac{x - vt}{1 - v^2/c^2}, \tag{17a}$$

$$\tau = a\frac{t - vx/c^2}{1 - v^2/c^2}. \tag{17b}$$

We include Eq. (17) only to highlight an ambiguity that followed. Einstein provided no explanation or justification for his next step; he did not even explicitly state it. Miller notes that "without prior warning," Einstein assigned a value for the undetermined term a, namely:

$$a = \varphi(v)\sqrt{1 - v^2/c^2}, \tag{18}$$

that is, $a = \varphi(v)b$ as stated above. This tacit step eliminated the denominators in the equations for η and ζ and introduced the term $\beta = 1/\sqrt{1 - v^2/c^2}$ into the equations for ξ and τ. Miller asked: "But why did Einstein make this replacement? It seems as if he knew beforehand the correct form of the set of relativistic transformations... ."[26] Certainly his first published derivation cannot be construed to be his first investigation of the problem. Miller offered a few historical conjectures to explain how Einstein found the final form of the transformations (in part from Einstein's study of Lorentz's paper of 1895, which, however, lacked the exact transformations), but such arguments need not be reviewed here. Einstein wanted to isolate the term $\varphi(v)$ to then show that it is irrelevant. But the important point is that Einstein did not introduce the value of a in the explicit manner in which he derived the values of the other terms. In view of this unsubstantiated step, readers of Einstein's derivation who did not have a predetermined notion of the final form of the transformation equations might well have been puzzled by his introduction of a and the β factor.

Finally, Einstein completed his derivation of the transformation equations by determining the value of the function $\varphi(v)$. He ascertained that $\varphi(v)=1$ by a long argument that involved the introduction of a third coordinate system (identical with K), and the measurement of the length of a moving rod oriented perpendicularly to its direction of motion. Again, we skip those arguments for the sake of brevity.

III. COMMENTARY AND CONCLUSION

Because we are focusing on the ambiguities in Einstein's expressions, consider again his use of the term x'. We raised the question of whether Einstein used a third coordinate system from the beginning of his analysis, and whether such a system was described by the laws of traditional kinematics. Presumably, Einstein introduced the auxiliary term x' to simplify his derivation, but did it really perform this function?

To clarify this issue, we reconstruct Einstein's derivation without introducing such a term, in the Appendix. There we confirm that the term x' was not essential for Einstein's analysis. His choice to express the dependence of the function τ on x' simplified the initial steps of his analysis, because it is a bit more complicated to differentiate τ with respect to x because the position of the light ray varies with the changing values of t. Instead of performing every calculation with the coordinates x_0, x_1, x_2, Einstein simply carried the auxiliary term x' throughout the derivation to reintroduce its value at the end. Nonetheless, it also is correct to evaluate the function τ with respect to x on an equal footing with the terms y, z, t, and, after differentiation, the procedure even leads more directly to the desired results, especially because it yields the value of the β factor directly, rather than requiring the introduction of the function $a=\varphi(v)b$.

The role of the term x' involves sufficient ambiguities to suggest that Einstein's derivation may have confused even careful readers of his paper, because even later physicists misinterpreted the role of x'. Because Eq. (4) is formally identical to a transformation equation, readers could easily misunderstand Einstein's analysis. For example, even Torretti misconstrued Eq. (4) as a transformation. Likewise, Miller misconstrued x' as a "Galilean coordinate" of system k, and accordingly, he misinterpreted the expressions $c-v$ and $c+v$ as speeds of light relative to k, as though Einstein had relied on the Galilean transformation of velocities in his analysis. He also construed Einstein as having employed x' to avoid discussing the relativistic effect of length contraction from the outset.[27] It would then seem that Einstein derived new transformation equations by assuming as true the traditional transformations and violating his own notion of the invariance of the speed of light. On such grounds, anyone who approached Einstein's analysis with a critical attitude could easily be confused or reject it as incoherent. But contrary to such interpretations, the term x' does not correspond to a Galilean or quasi-Galilean coordinate of system k. Instead, it may designate the constant separation between the two clocks as judged not from k but from K.

Yet even in this light, Einstein's analysis involved additional peculiarities that could engender confusion. For example, in Eq. (4) the term x' might seem to be the initial position of a material point in its equation of rectilinear motion. Also, Eq. (5) could be misinterpreted as consisting of coordinate values of k, whereas the arguments in each set of parentheses belong to K, and only three in each, namely the y, z, t terms, are coordinates. Einstein compounded this ambiguity by talking in one place about x' as if it were the position of the light ray when it reaches clock B. Such sloppiness of expression and notation was quite common in papers on theoretical physics at the time, but in the demonstration of radically novel claims, it could hardly help their intelligibility. Yet Einstein gave more than one meaning to some of the symbols he employed, for example, to x, x', and t. By avoiding exact expressions for the positions of a light ray along the X axis, his analysis scarcely distinguished between the concepts of position and length. For the most part, he distinguished explicitly the concepts of time intervals and instants (single time coordinates), but he hardly drew analogous distinctions for the concepts of space. Also, he didn't explicitly distinguish between concepts of distance and displacement, nor between speed and velocity; distinctions that stemmed from vector theory and had been introduced precisely to clarify the representation of physical quantities. Thus, Einstein's derivation focused on a clarification of the concept of time, on the basis of measurement procedures and algebraic analysis, but it admitted various ambiguities in the representation of other kinematic concepts.[28]

Moreover, his derivation suffers from some deficiencies. Consider again the factor β in the transformation equations. This factor is crucial, because it is a telltale feature that distinguishes the Lorentz transformation from the Galilean transformation. But Einstein did not derive this factor exactly, but introduced it rather freely by setting the value of the variable a to obtain the form of the transformations that he deemed to be correct. In addition to the β factor, the only other deviation from the Galilean transformations is the term $-xv/c^2$ in the time transformation. It seems awkward that most of the analysis in Einstein's long derivation simply results in the introduction of this small algebraic term (although physically the term is of crucial importance because it involves first-order effects in velocity v, whereas the β factor concerns second-order effects). Again, his use of differential analysis was an unnecessary detour. Thus the amount of analysis does not seem commensurate with the formal simplicity of its results.

Einstein's derivation lacked mathematical elegance; its degree of abstraction along with its ambiguities serve to illustrate why many of his contemporaries had difficulties understanding and accepting his kinematics. Back then, readers well-trained in theoretical physics constituted a small minority of physicists. Many steps of his derivation could seem debatable or imprecise to readers who took his every expression literally, or to those who did not already know or accept the final result, the transformation equations and their utility.

The structure and details of the derivation engender the impression of Einstein groping tortuously to construct it, having first discovered that the relativity of simultaneity could provide a kinematic justification for the transformations that he knew to work in optics and electromagnetism. To be sure, the general approach and novel concepts made his derivation well worthwhile. He demonstrated that the Lorentz transformations could be deduced from simple kinematic assumptions, along with the postulate of the constancy of the speed of light, irrespective of the exact validity of the rest of contemporary electromagnetic theory.

After the paper was published in the Annalen der Physik in 1905,[1] Einstein discarded the original manuscript.[29] Late in life, Einstein expressed surprise at the complexity of the paper. In 1943, Einstein was drafting a copy of the paper to donate for a fundraising auction. His secretary, Helen Dukas, recounted that "... she would sit next to Einstein and dictate the text to him. At one point, Einstein lay down his pen, turned to Helen and asked her whether he had really said what she had just dictated to him. When assured that he had, Einstein said, 'Das hätte sich einfacher sagen können.' "[30]

In comparison to other parts of Einstein's 1905 paper, his derivation of the transformation equations stands out as by far the most subtle mathematical argument. Later derivations of the same equations, by Einstein and others, were immensely simpler.[31] Hence we may understand Einstein's remark, "That could have been said more simply," as referring to his derivation of the transformation equations.

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APPENDIX: ALTERNATIVE DERIVATION OF THE TRANSFORMATIONS

We now reconstruct Einstein's derivation without introducing the x' term. Let us see what we derive instead of Eq. (5) if we establish the exact values of the coordinates in K at the times of emission, reflection, and return of the light ray traveling between the clocks in k. Consider again the expression, Eq. (8),

$$\tfrac{1}{2}[\tau(x_0,y_0,z_0,t_0)+\tau(x_2,y_2,z_2,t_2)]=\tau(x_1,y_1,z_1,t_1).$$

Each of the values of the coordinates in K can be established as follows. At the time t_0 if the origins of K and k coincide, and a light ray is emitted, its coordinates in K are $x_0=y_0=z_0=0$ and $t_0=t$. At this time the point ξ (where a clock is fixed to the Ξ axis) is located at a distance x from the origin of K. As the light ray travels along the X axis, ξ moves away with velocity ν. Once the ray traverses the distance x, the point ξ is no longer there, and the ray must traverse an extra distance $\nu(t_1-t_0)$ to reach ξ. Thus the coordinates in K of the signal upon its arrival at ξ are

$$x_1=x+\nu(t_1-t_0), \tag{19a}$$

$$y_1=0, \tag{19b}$$

$$z_1=0, \tag{19c}$$

$$t_1=t+(x/(c-\nu)). \tag{19d}$$

At this time t_1 the light signal and the point ξ coincide at a distance $x+\nu(t_1-t_0)$ from the origin of K. Now the ray travels in the opposite direction, while the origin of k moves toward it with the velocity ν. Because at the time t_1 the origin of k was at a distance x from the ray, the ray traverses less than this distance to meet it. Thus the coordinates in K of the ray when it reaches k's origin are

$$x_2=\nu(t_1-t_0)+\nu(t_2-t_1), \tag{20a}$$

$$y_2=0, \tag{20b}$$

$$z_2=0, \tag{20c}$$

$$t_2=t+(x/(c-\nu))+(x/(c+\nu)). \tag{20d}$$

These values can be restated by expressing (t_1-t_0) and (t_2-t_1) in terms of x, ν, and c:

$$(t_1-t_0)=x/(c-\nu), \tag{21a}$$

$$(t_2-t_1)=x/(c+\nu). \tag{21b}$$

Equations (21a) and (21b) are equivalent to Eqs. (6a) and (6b). So, the successive values of the x and t coordinates in K are

$$x_0=0, \tag{22a}$$

$$t_0=t, \tag{22b}$$

$$x_1=x+\nu x/(c-\nu), \tag{22c}$$

$$t_1=t+(x/(c-\nu)), \tag{22d}$$

$$x_2=\nu x/(c-\nu)+\nu x/(c+\nu), \tag{22e}$$

$$t_2=t+(x/(c-\nu))+(x/(c+\nu)). \tag{22f}$$

These values can be simplified but the above form conveys their physical significance. These values can now be substituted into Eq. (8), so that we obtain:

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(\frac{\nu x}{c-\nu}+\frac{\nu x}{c+\nu},0,0,t+\frac{x}{c-\nu}+\frac{x}{c+\nu}\right)\right]$$
$$=\tau\left(x+\frac{\nu x}{c-\nu},0,0,t+\frac{x}{c-\nu}\right), \tag{23}$$

instead of Einstein's expression, Eq. (5),

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,t+\frac{x'}{c-\nu}+\frac{x'}{c+\nu}\right)\right]$$
$$=\tau\left(x',0,0,t+\frac{x'}{c-\nu}\right).$$

Are the two expressions equivalent? Einstein's formulation is algebraically simpler, because of his use of x', but conceptually it is ambiguous for the reasons mentioned. To further distinguish the two approaches, we seek the difference between the next result Einstein obtained and what follows otherwise.

To obtain the differential equation expressing the dependence of τ on x and t, as in Einstein's approach, we may make a series expansion of each term in τ. For example, because

$$x+\frac{\nu x}{c-\nu}=x\frac{c}{c-\nu}, \tag{24}$$

then for τ_1:

$$\tau(xhc,t+hx)=\tau(xhc,t)+\frac{\partial\tau}{\partial t}\frac{hx}{1}+\frac{\partial^2\tau}{\partial t^2}\frac{(hx)^2}{2!}+\cdots, \tag{25}$$

where we have let $h=1/(c-\nu)$. Now, if we differentiate the function and the series with respect to x, we obtain

$$\frac{\partial\tau(xhc,t+hx)}{\partial x}=\frac{\partial\tau}{\partial x}hc+\frac{\partial\tau}{\partial t}h+\frac{\partial^2\tau}{\partial t^2}xh^2+\cdots \tag{26}$$

If we neglect terms of second order and higher, we find

$$\frac{\partial\tau_1}{\partial x}=\frac{\partial\tau}{\partial x}\left(\frac{c}{c-\nu}\right)+\frac{\partial\tau}{\partial t}\left(\frac{1}{c-\nu}\right). \tag{27}$$

By carrying out the same procedure for the other side of Eq. (23), we obtain

$$\frac{1}{2}\left[(0)+\frac{\partial\tau}{\partial x}\left(\frac{2\nu c}{c^2-\nu^2}\right)+\frac{\partial\tau}{\partial t}\left(\frac{2c}{c^2-\nu^2}\right)\right]$$
$$=\frac{\partial\tau}{\partial x}\left(\frac{c}{c-\nu}\right)+\frac{\partial\tau}{\partial t}\left(\frac{1}{c-\nu}\right), \tag{28}$$

so that we now obtain

$$\frac{\partial\tau}{\partial x}+\frac{\nu}{c^2}\frac{\partial\tau}{\partial t}=0. \tag{29}$$

Compare this result to Einstein's:

$$\frac{\partial \tau}{\partial x'} + \frac{v}{c^2 - v^2} \frac{\partial \tau}{\partial t} = 0. \tag{30}$$

From Eq. (30) he obtained the algebraic expression for τ, Eq. (15). But following the same procedure we derive from Eq. (29) the different equation:

$$\tau = \beta(t - vx/c^2) \tag{31}$$

(where the value of β has not yet been determined), instead of Einstein's result, Eq. (15),

$$\tau = a\left(t - \frac{v}{c^2 - v^2} x' \right),$$

which may be restated by including the value he gave to x', that is, $x' = x - vt$:

$$\tau = a\left(t - \frac{vx - v^2 t}{c^2 - v^2} \right). \tag{32}$$

Notice how Eq. (31) resembles the transformation equation for τ that Einstein only subsequently obtained: $\tau = \beta(t - vx/c^2)$.

The only distinction between Eq. (31) and the final transformation is that we have yet to establish that β has the same value in both. This great similarity indicates that, for this point onward, the algebraic derivation of the τ function in terms of x instead of x' leads more directly to the results sought by Einstein. All that remains is to show that this approach agrees with Einstein's by completing the derivation of the transformation equations. Again, we will proceed in direct analogy to Einstein's 1905 procedure.

To express the quantities ξ, η, ζ in terms of x, y, z, we need to express in equations that light propagates with a speed c when measured in the moving system. For a light ray emitted at the time $\tau = 0$ in the direction of increasing ξ, $\xi = c\tau$, so that from Eq. (31) we obtain:

$$\xi = \beta(ct - vx/c). \tag{33}$$

Because x corresponds to the distance light travels in K in a time t, $x = ct$, so that

$$\xi = \beta(x - vt). \tag{34}$$

To obtain the equations relating η and ζ to y and z, we proceed in an analogous manner by considering rays moving along the two other axes. First, for the H axis:

$$\eta = c\tau = \beta c(t - vx/c^2). \tag{35}$$

Now, *as observed from the system K*, a ray of light propagating along the H axis of k travels a diagonal path with a speed $\sqrt{c^2 - v^2}$, such that when

$$\frac{y}{\sqrt{c^2 - v^2}} = t, \quad x = vt, \tag{36}$$

because the H axis has moved a distance vt as the ray travels to the point η. If we substitute this value of x into the expression for η, Eq. (35), we find:

$$\eta = \beta ct\left(1 - \frac{v^2}{c^2} \right). \tag{37}$$

We substitute the value of t to find:

$$\eta = \beta y \sqrt{1 - v^2/c^2}. \tag{38}$$

The same procedure for the light ray transmitted along the Z axis yields an equivalent result, so that the resulting four equations are

$$\tau = \beta(t - vx/c^2), \tag{39a}$$

$$\xi = \beta(x - vt), \tag{39b}$$

$$\eta = \beta y \sqrt{1 - v^2/c^2}, \tag{39c}$$

$$\zeta = \beta z \sqrt{1 - v^2/c^2}. \tag{39d}$$

Equation (39) now suggests that $\beta = 1/\sqrt{1 - v^2/c^2}$, which may be demonstrated by using the same arguments Einstein used to establish that $\varphi(v) = 1$, or any simpler arguments showing, say, that $\eta = y$. Hence we arrive again at the final form of the transformation equations.

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[1]A. Einstein, "Zur Elektrodynamik bewegter Körper," Ann. Phys. (Leipzig) **17**, 895–921 (1905); reprinted in *The Collected Papers of Albert Einstein*, edited by John Stachel (Princeton U.P., Princeton, 1989), Vol. 2. Several translations of Einstein's paper are available, including the following. H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity*, translated by W. Perrett and G. B. Jeffery with notes by Arnold Sommerfeld (Dover, New York, 1952). Anna Beck (translator), and Peter Havas (consultant), *The Collected Papers of Albert Einstein* (Princeton U.P., Princeton, 1989), Vol. 2. Arthur I. Miller, *Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905–1911)* (Addison–Wesley, Reading, MA, 1981) and (Springer-Verlag, New York, 1998). *Einstein's Miraculous Year; Five Papers That Changed the Face of Physics*, edited by John Stachel, with the assistance of Trevor Lipscombe, Alice Calaprice, and Sam Elworthy (Princeton U.P., Princeton, 1998). Note that the translations by Perrett and Jeffery and by Miller have some slight but significant defects.

[2]The quotes that follow are from Carl Seelig, *Albert Einstein: A Documentary Biography*, translated by Mervyn Savill (Staples, London, 1956), pp. 73, 88. See also Max Flückiger, *Albert Einstein in Bern: Das Ringen um ein neues Weltbild* (Paul Haupt, Berne, 1974), pp. 103, 209.

[3]Russell McCormmach, "Editor's Foreword," *Historical Studies in the Physical Sciences*, 7th Annual Volume (Princeton University Press, Princeton, 1976), p. xxxvi. McCormmach's expression was later modified to read "ordinary algebra ... modest mathematical equipment," in Christa Jungnickel and Russell McCormmach, *Intellectual Mastery of Nature* (University of Chicago Press, Chicago, 1986), Vol. 2, p. 337.

[4]For example, even William Rowan Hamilton, with his extraordinary mathematical skills, encountered difficulties as a youth studying Newton's works; he commented that Newton wrote the *Universal Arithmetick* in "the same masterly manner as the *Principia*, and yet in many parts is rendered almost as difficult, by its conciseness and omission of intermediate steps." Letter of 28 September, 1823, in W. R. Hamilton, *Life of Sir William Rowan Hamilton*, edited by R. P. Graves (Hodges, Figgis & Co., Dublin, 1882), Vol. 1, p. 149.

[5]For example, see Miller in Ref. 1, Torretti in Ref. 14, or Robert B. Williamson, "Logical economy in Einstein's 'On the Electrodynamics of Moving Bodies,' " Stud. Hist. Philos. Sci. **8**, 46–60 (1977), or Harvey R. Brown and Adolfo Maia, Jr., "Light-speed constancy versus light-speed invariance in the derivation of relativistic kinematics," Br. J. Philos. Sci. **44**, 381–407 (1993).

[6]Miller, Ref. 1.

[7]H. A. Lorentz, "De l'influence du mouvement de la terre sur les phénomènes lumineux," Versl. Gewone Vergad. Afd. Koninklijke Akademie van Wetenschappen te Amsterdam **2**, 297–372 (1886); also in Archives Néerlander **21**, 19–84 (1887); reprinted in H. A. Lorentz, *Collected Papers* (Martinus Nijhoff, The Hague, 1937), Vol. 4, pp. 153–214.

[8]H. A. Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern* (Brill, Leiden, 1895); reprinted in H. A. Lorentz, *Collected Papers*, Vol. 5, pp. 1–137. Joseph Larmor, *Aether and Matter: A Development of the Dynamical Relations of the Aether to Material Systems on the Basis of the Atomic Constitution of Matter; including a Discussion of the Influence of the Earth's Motion on Optical Phenomena* (Cambridge U.P., Cambridge, 1900). H. A. Lorentz, "Electromagnetische

verschijnselen in een stelsel dat zich met willekeurige snelheid, kleiner dan die van het licht, beweegt,'' Koninklijke Akademie van Wetenschappen te Amsterdam. Wis-en Natuurkundige Afdeeling. Verslagen van de Gewone Vergaderingen **12**, 986–1009 (1904); reissued as ''Electromagnetic phenomena in a system moving with any velocity smaller than that of light,'' Koninklijke Akademie van Wetenschappen te Amsterdam, Section of Sciences, Proceedings **6**, 809–831 (1904). Henri Poincaré, ''Sur la dynamique de l'électron,'' Comptes rendus de l'Académie des Sciences [Paris] **40**, 1504–1508 (1905); reprinted in H. Poincaré, Oeuvres **9**, 489–493 (1954). Henri Poincaré, ''Sur la dynamique de l'électron,'' Rendiconti del Circolo matematico di Palermo **21**, 129–176 (1906); reprinted in Oeuvres **9**, 494–550 (1954).

[9]W. Voigt, ''Ueber das Doppler' sche Princip,'' Königliche Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen. Nachrichten **14**, 41–51 (1887); reprinted in Phys. Z. **16**, 381–386 (1915).

[10]P. Frank, ''Die Stellung des Relativitätsprinzips im System der Mechanik und der Elektrodynamik,'' Sitzungsber. Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Klasse [Wien] Section IIa **118** (4), 373–446 (1909).

[11]Einstein, Ref. 1, p. 898; italics in the original.

[12]For a rigorous discussion of the connection between homogeneity and linearity, see Roberto Torretti, *Relativity and Geometry* (1983), reprinted in (Dover, New York, 1996), pp. 71–76. Torretti systematically clarifies the matter because he notes that at first ''it is not at all clear that the linearity of the Lorentz transformations can follow from this fact alone'' (p. 72), the homogeneity of space and time. Torretti shows that the linearity can be justified on physical grounds by appealing to the principle of inertia. Also, Olivier Darrigol suggests (personal communication) that by homogeneity Einstein presumably meant that a straight line should transform into a straight line, and that a rectilinear uniform motion should transform into another of the same kind.

[13]Einstein, Ref. 1, p. 898.

[14]Equation (4) has been identified as a transformation; see, for example, Torretti, Ref. 12, p. 58.

[15]See for instance H. A. Lorentz, *Lehrbuch der Differential- und Integralrechnung nebst einer Einführung in andere Teile der Mathematik*, revised by G. C. Schmidt (Barth, Leipzig, 1900), p. 83.

[16]Miller, Ref. 1, p. 196; see also p. 213. The claim that x', y', z' are coordinates of the moving system also is made in Stachel (1998), Ref. 1, p. 160, Note 2.

[17]Torretti, Ref. 12, pp. 57–58.

[18]Gustav Kirchhoff, *Vorlesungen über Mathematische Physik: Mechanik* (Teubner, Leipzig, 1883), 3rd ed., p. 4.

[19]Einstein, Ref. 1, p. 898.

[20]Miller, Ref. 1, p. 397.

[21]Miller, Ref. 1, p. 209. Moreover, throughout the decades some writers mistakenly criticized Einstein's demonstration of the relativity of simultaneity and his derivations of the Lorentz transformations claiming that Einstein had violated his postulate of the constancy of the speed of light by employing speeds other than c in his arguments.

[22]Some thorough books do not neglect this important distinction. See, for example, Henri Arzèlies, *Relativistic Kinematics* (Pergamon, Oxford, 1966), pp. 142–143.

[23]Miller, Ref. 1, p. 209.

[24]I thank Olivier Darrigol for elucidating this account.

[25]Einstein, Ref. 1, p. 899.

[26]Miller, Ref. 1, p. 212. Likewise, Fölsing comments: ''At an opaque point in his deduction he [Einstein] introduces, without any warning or explanation, a slight mathematical operation whose purpose becomes obvious only if the desired result is already known. This underhand device, by means of which he rather forcibly 'computes his way' to the Lorentz transformations, deprives the deduction of some of its elegance and stringency.'' Albrecht Fölsing, *Albert Einstein. A Biography*, translated by Ewald Osers (Viking/Penguin, New York, 1997), p. 188.

[27]Miller, Ref. 1, p. 209.

[28]Alberto A. Martínez, ''The Neglected Science of Motion: The Kinematic Origins of Relativity,'' Ph.D. dissertation, University of Minnesota, December, 2000, UMI, 2001.

[29]As Einstein informed Julian Boyd, librarian of the Princeton University Library. See Abraham Pais, *'Subtle is the Lord...' The Science and the Life of Albert Einstein* (Oxford U.P., New York, 1982), p. 147. Einstein's declaration of having discarded the original manuscript appears in Flückiger, Ref. 2, p. 103.

[30]Pais, Ref. 29, p. 147. John Stachel, having also heard the story directly from Helen Dukas, received the impression that it was at two distinct places during the reading that Einstein made this remark to her. We do not know whether he necessarily referred to something in his derivation of the transformations. Incidentally, the *Collected Papers of Albert Einstein*, Vol. 2 (Ref. 1, p. 309) includes notes on three slight corrections that Einstein made to a reprint copy of his 1905 paper, but such corrections are far too minor for us to assume that his later comment to Dukas referred to them.

[31]For example: A. Einstein, ''Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen,'' Jahrbuch der Radioactivität und Elektronik **4**, 411–462 (1907); reprinted in *The Collected Papers of Albert Einstein*, edited by John Stachel (Princeton U.P., Princeton, 1989), Vol. 2; and A. Einstein, *Relativity: The Special and General Theory*, translated by Robert W. Lawson (P. Smith/Henry Holt and Co., New York, 1931), Appendix I.